Inequality with angle bisectors, exstanded angle bisectors and sinuses.
https://www.linkedin.com/feed/update/urn:li:activity:6635213827829039104
In common notation:
Let ABC be a triangle inscribed in a circle and let l_a, l_b, l_c be lengths of the angle bisectors (internal to the triangle) and let L_a, L_b, L_c be lengths of the angle bisectors extended until the meet the circumcircle. Prove that

$$\frac{l_a}{L_a \sin^2 A} + \frac{l_b}{L_b \sin^2 B} + \frac{l_c}{L_c \sin^2 C} \geq 3.$$

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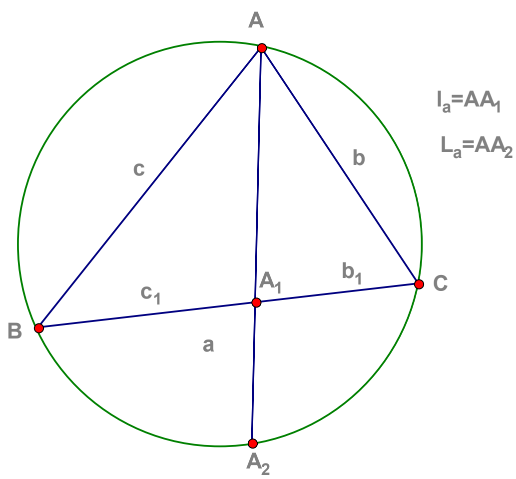

Let R, r and s be, respectively, circumradius, inradius and semiperimeter of $\triangle ABC$.
By Intersecting Chord Theorem we have $AA_1 \cdot A_1A_2 = BA_1 \cdot A_1C \Leftrightarrow$ $l_a(L_a - l_a) = b_1c_1 \Leftrightarrow l_aL_a = b_1c_1 + l_a^2$. Since $l_a^2 = bc - b_1c_1$ then $l_aL_a == bc$ and, therefore, $\dfrac{l_a}{L_a} = \dfrac{l_a^2}{l_aL_a} = \dfrac{l_a^2}{bc}$. Hence, noting that $a = 2R\sin A$ and $abc = 4Rrs$ we obtain $\sum \dfrac{l_a}{L_a \sin^2 A} \geq 3 \Leftrightarrow \sum \dfrac{l_a^2}{bc\sin^2 A} \geq 3 \Leftrightarrow \sum \dfrac{l_a^2}{a^2bc} \geq \dfrac{3}{4R^2} \Leftrightarrow$
$\sum \dfrac{l_a^2}{a} \geq \dfrac{3abc}{4R^2} \Leftrightarrow \sum \dfrac{l_a^2}{a} \geq \dfrac{3rs}{R}$.

Since by Cauchy Inequality $\sum \dfrac{l_a^2}{a} \geq \dfrac{\left(\sum l_a\right)^2}{a+b+c} = \dfrac{1}{2s}\left(\sum l_a\right)^2$ and $l_x \geq h_x, x \in \{a,b,c\}$ then $\sum \dfrac{l_a^2}{a} \geq \dfrac{1}{2s}\left(\sum h_a\right)^2$ and noting that $h_a = \dfrac{bc}{2R}$ we obtain $\sum \dfrac{l_a^2}{a} \geq \dfrac{(ab+bc+ca)^2}{8sR^2}$.
Also we have $(ab+bc+ca)^2 \geq 3abc(a+b+c) = 3 \cdot 4Rrs \cdot 2s = 24Rrs^2$.
Thus, $\sum \dfrac{l_a^2}{a} \geq \dfrac{24Rrs^2}{8sR^2} = \dfrac{3rs}{R}$.

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